Today’s post is again focused on arithmetic mean. Let’s start our discussion by considering the case of arithmetic mean of an arithmetic progression.

We will start with an example. What is the mean of 43, 44, 45, 46, 47? (Hint: If you are thinking about adding the numbers, that’s not the way I want you to go.)

As we discussed in our previous posts, arithmetic mean is the number that can represent/replace all the numbers of the sequence. Notice in this sequence, 44 is one less than 45 and 46 is one more than 45. So essentially, two 45s can replace both 44 and 46. Similarly, 43 is 2 less than 45 and 47 is 2 more than 45 so two 45s can replace both these numbers too.

The sequence is essentially 45, 45, 45, 45, 45.

Hence, the arithmetic mean of this sequence must be 45! (If you have doubts, you can calculate and find out.)

It makes sense, doesn’t it? The middle number in the sequence of consecutive positive integers will be the mean. The deviations of all numbers to the left of the middle number will balance out the deviations of all the numbers to the right of the middle number.

(In this post, we will assume that the given numbers are in increasing/decreasing order. If that is not the case, you can always put them in increasing order and use these concepts.)

Once again, what is the mean of 192, 193, 194, 195, 196, 197, 198?

It is 195 since it is the middle number!

Ok, what about 192, 193, 194, 195, 196, 197? What is the mean in this case? There is no middle number here since there are 6 numbers. The mean here will be the middle of the two middle numbers which is 194.5 (the middle of the third and the fourth number). It doesn’t matter that 194.5 is not a part of this list. If you think about it, arithmetic mean of some numbers needn’t be one of the numbers.

What about 71, 73, 75, 77, 79? What will be the mean in this case? Even though these numbers are not consecutive integers, the difference between two adjacent numbers in the list is the same (it is an arithmetic progression). So the deviations of the numbers on the left of the middle number will cancel out the deviations of the numbers on the right of the middle number (71 is 4 less than 75 and 79 is 4 more than 75. 73 is 2 less than 75 and 77 is 2 more than 75). Hence, the mean here will be 75 (just like our first example).

Just to re-inforce:

102, 106, 110 –> Mean = 106

102, 106, 110, 114 -> Mean = 108 (Middle of the second and third numbers)

Let’s twist this concept a little now. What is the mean of 36, 40, 42, 43, 44, 47?

This is not an arithmetic progression. So do we need to sum and then divide by 6 to get the mean? Not so fast! Let’s try and use the deviations concept we have just learned.

Given sequence: 36, 40, 42, 43, 44, 47

It seems that the mean would be around 42, right? Some numbers are less than 42 and others are more than 42.

36 is 6 less than 42.

40 is 2 less than 42.

Overall, the numbers less than 42 are 6+2 = 8 less than 42.

43 is 1 more than 42.

44 is 2 more than 42.

47 is 5 more than 42

Overall, the numbers more than 42 are 1+2+5 = 8 more than 42.

The deviations of the numbers less than 42 get balanced out by deviations of the numbers greater than 42! Hence, the average must be 42.

This method is especially useful in cases involving big numbers which are close to each other.

Example 1: What is the average of 452, 453, 463, 467, 480, 499, 504?

What would you say the average is here? Perhaps, around 470?

Let's see:

452 is 18 less than 470.

453 is 17 less than 470.

463 is 7 less than 470.

467 is 3 less than 470.

Overall, the numbers less than 470 are 18 + 17 + 7 + 3 = 45 less.

480 is 10 more than 470.

499 is 29 more than 470.

504 is 34 more than 470.

Overall, the numbers more than 470 are 10 + 29 + 34 = 73 more than 470.

The shortfall is not balanced by the excess. There is an excess of 73 – 45 = 28.

So what is the average? If we assume the average of these 7 numbers to be 470, there is an excess of 28. We need to distribute the excess evenly among all the numbers and hence the average will increase by 28/7 = 4. (Go back tothe first post on arithmetic mean if this is not clear.)

Hence, the required mean is 470 + 4 = 474.

(If we had assumed the mean to be 474, the shortfall would have balanced the excess.)

Let's go through one more example using this concept:

Example 2: What is the mean of 99, 103, 104, 109, 120, 123, 128, 130?

Let's start by guessing a mean for this sequence. Say, around 115?

Let's see if the shortfall is balanced by the excess.

99 is 16 less, 103 is 12 less, 104 is 11 less and 109 is 6 less than 115.

Overall shortfall = 16 + 12 + 11 + 6 = 45

120 is 5 more, 123 is 8 more, 128 is 13 more and 130 is 15 more than 115.

Overall excess = 5 + 8 + 13 + 15 = 41

We are close, but not quite there yet! There is a shortfall of 4. Since there are a total of 8 numbers, the average must be 4/8 = 0.5 less than 115. Hence, the average here is 114.5

Once you get a hang of this method and understand what you are doing, it is much faster than adding all the big numbers and then dividing the sum since you only deal with small numbers in this method.

Let's wrap up today's post here. Next week, we will see these concepts in action!